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Material Mediation: Tools and Representations Supporting Collaborative Problem-Solving Discourse

Elvira K. Katić
Ramapo College of New Jersey

Cindy E. Hmelo-Silver and Keith H. Weber
Rutgers University

This study investigates how a variety of resources mediated collaborative problem solving for a group of preservice teachers. The participants in this study completed mathematical, combinatorial tasks and then watched a video of a sixth grader as he exhibited sophisticated reasoning to recognize the isomorphic structure of these problems. The preservice teachers used a variety of material tools to solve the same problem, construct explanations of the learning processes that the sixth grader engaged in, and pose further questions about the problem to clarify their solutions. The results of this study suggest that simple material tools helped to motivate and mediate the participants' collaborative problem-solving discourse.

Since Vygotsky (1978) described the importance of mediation to situated learning, researchers have been examining the interplay among agents, tools, and activities (Cole, 1996; Engeström, 1999). Because tools play such an important role in sociocultural theories of learning, our research question focuses on how tools mediate collaborative learning in a higher education setting. Tools both guide and constrain learning activities by allowing learners to engage in some kinds of activities and preventing them from engaging in others (Pea, 2004). This is particularly important as teacher education programs and higher education institutions in general move to collaborative and learner-centered models of teaching (e.g., Ball & Wells, 2006; Darling-Hammond & Hammerness, 2005; Herrington & Herrington, 2006), such as problem-based learning. Problem-based learning is a collaborative, student-centered approach to instruction in which students learn through solving problems (Hmelo-Silver, 2004). In these instructional models, the teacher serves as a facilitator of learning, often working with multiple groups, and the tools available in the environment serve to support student learning and activity (Greeno, Collins, & Resnick, 1996; Hmelo-Silver, Duncan, & Chinn, 2007). Despite the importance of tools in such environments, and much theory about how they should mediate learning, there is not a lot of research on the role of tools in collaborative learning.

This research examines how a group of preservice teachers used different tools as resources for mediating collaborative discourse while engaged in a problem-solving task. In the problem-based learning classroom we observed, each preservice teacher group was asked to work together to solve a mathematical problem and later analyze a videocase of a middle-school student solving the same problem. We describe how one group used a variety of different tools during their problem solving as contribution to the research on mediational tools in collaborative learning and problem solving in higher education. This is an important issue as problem-based learning and other collaborative, student-centered approaches are being increasingly used in diverse higher education settings (e.g., Herrington & Herrington, 2006; Major, Savin-Baden, & MacKinnon, 2000; te Winkel, Rikers, & Schmidt, 2006). Our research questions are twofold. The primary question is how material tools and artifacts mediate collaborative learning. As we began investigating this question, student's beliefs about the way that he or she used these tools and artifacts emerged as a secondary question.

Material Representations and Mediation

Cultural artifacts and representations are tools that people can modify to regulate their goal-directed activities (Cole, 1996; Engeström, 1999; Pea, 1993). As these tools organize and constrain human activity, they can help structure people's thinking and action. Our focus in this study is on designed artifacts (such as manipulatives) and written representations (such as diagrams). Such use of material mediational tools allows difficult and elaborate reasoning tasks to be distributed into the physical environment.

Mediation is one of the basic principles of cultural historical activity theory (Cole, 1996). Tools are artifacts or representations that can be used to modify human activity. They may be either external (such as a poster or a computer) or internal (such as language) mediators. Tools exert a strong influence on physical and mental operations. Finally, tools serve a communicative purpose and can be used by individuals to exchange knowledge with their peers. Thus, tools and representations are more than just inert paraphernalia. They are imbued with cultural meaning and become key mediators that partially direct resulting human actions (Engeström, 1999). Researchers have demonstrated the important role of tools in mediating problem solving. For example, in a study with middle school children, Barron (2003) found that artifacts in the form of workbooks served as centers for coordinating collaborative mathematical problem-solving. Stevens (2000) analyzed the affordances of paper and computer-based tools for supporting collaboration in both middle school children and professional architects. He explored different forms of collaboration, asked why particular tools were used in particular ways, and found that paper tools afforded participants greater means to creativity, flexibility, and availability (among other affordances) over technological tools. We focus here on the tools and representations that figured in one group's collaborative problem-solving processes during work on two related tasks. We were interested in what tools were at the group's disposal, how they were used, and how they served to mediate collaborative problem-solving processes. This is a particular issue in teacher education because many teacher education pedagogies, including the problem-based learning approach in this study, are designed to help preservice teachers build the tools and practices needed for close analysis of teaching and learning (Darling-Hammond & Hammerness, 2005).

Relationships Between Shifts in Tool Use and New Mathematics Activity

Mathematics educators have documented that changes in the way students use mathematical tools can afford students the opportunity to participate in new mathematical activities. Further, in some cases, the new activities that students participate in may be critical in advancing students' mathematical thinking (e.g., Gravemeijer, 1999; Rasmussen, Zandieh, King, & Teppo, 2005). One such distinction is Gravemeijer's (1999) model of/model for dichotomy. At first, students may use physical tools (e.g., manipulatives) or written tools (e.g., a graph) to model a mathematical situation. At this stage, students are using these tools as a representation system for the purposes of understanding and description; the tools are used as a model of students' mathematical activity. Subsequently, students may view representation systems that they produced as interesting mathematical objects in their own right, and they may proceed to investigate the systems' mathematical properties. Students regard the tools they are using as a model for mathematical investigation. The shift from viewing tools as representing mathematical situations to becoming mathematical objects worthy of investigation in their own rights affords students the opportunity to engage in mathematical abstraction.

Similarly, when students engage in mathematical activity, they usually write inscriptions to mediate their work. Initially, these inscriptions may aid learners in completing a particular task, serving functions such as objectifying important mathematical ideas, assisting in calculations and manipulations, and reminding the students of what findings have been established (Harel & Kaput, 1991). Later, these inscriptions may be used as records of their mathematical activity, signifying mathematical activity that transpired (Cobb, Boufi, Whitenack, & McClain, 1997; Rasmussen & Marrongelle, 2006). These shifts enable participants to objectify their previous activity and make it an explicit subject of discussion, affording students new opportunities to justify the validity of the solutions they obtained (Cobb et al., 1997) and to advance their mathematical thinking by considering more abstract mathematical ideas (Rasmussen, Zandieh, Teppo, & King, 2005; Rasmussen & Marongelle, 2006). We will illustrate how one group of preservice teachers' experience with Unifix cubes in problem-solving and their observations of a student using these manipulatives led them to appreciate the mathematical and pedagogical values of these tools.

Tasks Used in This Study

The tasks used in this study are well researched in mathematics education as part of a longitudinal study of children's mathematical reasoning (Maher & Martino, 1996, 2001; Martino & Maher, 1999). The first task asks students to try to find all the possible ways to build towers with yellow and blue blocks that are four blocks tall and to justify that they have found all possible combinations. The second task asks students to list all the possible pizzas that could be ordered if four toppings were available and, again, to provide a justification that all combinations were produced. These two problems are isomorphic, as they share the same deep mathematical structure. For the first task, there are two choices for each block (blue or yellow), there are four such decisions to be made (one for each of the four blocks in the tower), and each of these decisions can be treated independently. Hence, there are $2^4 = 16$ possible towers. Similarly, for the second task, for each pizza topping, there are two choices that can be made (include the topping or not), there are four decisions to be made (one for each topping), and each decision is independent. Again, there are $2^4 = 16$ possible pizzas. In prior research, children's use of representations moved from strategies embedded in concrete artifacts to a later emphasis on more abstract, written representations (Maher & Martino, 1996). Earlier research demonstrated that as students connected representations, they reorganized their thinking and were able to construct the isomorphisms between the problems.

Methodology

Setting and Participants

The setting for this study was a semester-long course at the graduate school of education at a large, northeastern, United States university in 2003. The goal of the course was to have preservice teachers understand how learning principles applied to different types of classroom practice. Key participants in this research were the six students (2 male, 4 female) of Group 5: Bob, Caitlin, Liz, Helen, Carla, and Matt (pseudonyms). The students were all enrolled in a problem-based educational psychology course for preservice teachers. They were all between the ages of 20 and 30, Caucasian, and working on completing their respective teaching degrees. The students' major areas of study were varied, but none were Math Education majors.

First, students had to work on solving mathematical proof problems on the mathematical topic of combinations. They needed to prove that they had determined all of the possible combinations that could be created using two different color blocks in four-block-high towers. During one class session, the participants were provided with Unifix cubes in two colors (yellow and blue), and they needed to figure out how many differently patterned, four-cube-high towers they could create using the cubes. The students were clustered together in a circle, seated at individual desks, discussing the "block problem" (i.e., how many four-tall towers can one form with blue and yellow blocks?). Each of the six group members had a number of the Unifix cubes on their individual desks as they tried to justify they had identified all possible combinations that were four blocks high. Some group members manipulated the stacking blocks; others did not. As a group, they talked to one another and asked questions about different mathematical explanations that provided a proof.

In a subsequent class session, the group had to solve a related problem where they had to determine the number of different pizzas that could be made with four different topping options. In this exercise, the group was not provided with any manipulatives, such as blocks or plastic pizza shapes to work with. Although the block problem and the pizza problem were isomorphic, the group members did not use an analogous strategy to solve the pizza problem as they used to solve the block problem. For example, the students did not seek out or create concrete representations of the four different toppings and play around with different pizza combinations. Rather, the students used their knowledge of algebra to pose that 2^n n-tall towers could be formed with yellow and blue blocks and that 2^n pizzas could be created if there were *n* toppings to choose from. Three of the students seemed to have a partial understanding of this formula, but the remainder of the group had trouble grasping the connection between the algebraic formula and the blocks on the table in front of them.

After engaging in their own problem solving with the block and pizza problems, the group studied a videocase of a sixth grader, Brandon, solving the same pizza problem using stacking blocks and a chart that he had constructed. The group needed to analyze Brandon's thinking and identify the learning and reasoning strategies he employed while engaged with the mathematical problems (Maher, 1998). The group had several tools as available resources during problem-solving: 1) plastic stacking blocks (Unifix cubes) in blue and yellow, 2) adhesive paper whiteboards, 3) markers, 4) transcripts of Brandon and another child interacting while working on the pizza problem in class, and 5) a computer simulation of the block problem.

Data Collection

Methods of data collection included 14 hours of digital video of the group (eight class sessions) and transcripts from stimulated recall interviews with two of the participants. The primary form of data collection was the digital video from four of Group 5's class sessions in which the preservice teachers were working on the mathematics problems. Once the video was catalogued, sessions were reviewed with emphasis paid to moments that pointed to patterns of effective collaboration skills or use of various materials. The video was reviewed again to identify short clips of significant moments that ideally portrayed the following research concerns: 1) group work with shared representational media (i.e., blocks, paper whiteboards, posters) and 2) ways in which the use of mediating materials made evident some sensible interactions, patterns, or meaning(s) within group collaboration. Six clips (ranging from 1 to 3 minutes long) were selected and then transcribed in preparation for coding.

Stimulated recall interviews (Shavelson, Webb, & Burstein, 1986; Fontana & Frey, 2000) focused on obtaining the participants' explanations for what was captured in each of the significant clips. The protocol for the interview was inductive and consisted of open-ended questions designed to elicit descriptive responses from the participants regarding aspects of the significant clips. While core questions were identical for participants to compare different responses to the same prompt, probe questions varied in order to allow participants to express their thoughts about the different clips. Participants were questioned individually while viewing the video clips taken of their group work. These clips were played, one at a time, by each participant. Participants were encouraged to pause, rewind, or stop the video during discussion of the clip and protocol questions. Each participant was encouraged to reflect on the segment in his or her own words, and all discussion for a particular clip ended before moving to the next clip.

There are several benefits of the stimulated-recall interview format. One is that participants are able to "re-live" events that may have occurred some time in the past. They are able to pause time (in a sense) and reflect on a particular moment or closely examine events by manipulating the data medium (Bloom, 1953; Calderhead, 1981; Cresswell, 1998). In addition, both the interviewee and the interviewer are able to stay closer to the actual events, as opposed to asking questions removed from the event in both space and time: "Data elicited in this manner are likely to have greater ecological validity…more readily applicable to real conditions of work that data generated under more artificial circumstances" (Jordan & Henderson, 1995, p. 50).

Data Analysis

Analysis of the data was achieved through grounded theory methodology (Strauss & Corbin, 1998) in order to determine themes latent in the corpus of the video (as well as the discourse) data. Using grounded theory techniques, we began by dividing all of the data into episodes—short stretches of participant interaction that were bound by a common thread or topic. After episodes were identified, they were broken into smaller codable units—turns (within the episodes). Turns were coded using grounded theory methodology to facilitate the building of descriptive and dominant themes (Strauss & Corbin, 1998). Both the video and audio transcripts were thus divided into episodes, examined by turns, and coded dispassionately (i.e., without a priori labeling) but with an alert discernment for various types of tool use by the participants, discussion about tools and mediation, and evidence of social and/or material mediation as affecting the group's problem-solving discourse. The coded data turns that resulted from this analysis were then organized in terms of themes. The formulations of our hypotheses concerning the role(s) of material mediators occurred before, during, and after data collection and analysis. We constantly compared the similarities and differences of coded data in order to distill the data corpus into different categories. Portions or instances of data that did not directly relate to research concerns were eliminated (i.e., turns coded to indicate social talk) in order to reduce the data. Certain themes were collapsed into the same dominant category if their key components described the same basic observation or behavior.

The following is a data excerpt from a stimulated recall interview with Caitlin that took place on 8/21/03. This dialogic exchange was categorized as an episode, as it was a short stretch of participant interaction bound by a common thread or topic:

> Elvira: That's very interesting because you, you know, you said because you didn't really use the blocks at all.
>
> Caitlin: Yeah, so maybe, you know maybe I was using them and not even realizing it (She is speaking while watching clip 3 on the computer)... Yeah, no that was probably definitely...'cause I mean, I can see from…from looking at this (watching the clip and pointing to herself on the computer monitor), I was definitely using the tools, maybe I just didn't even realize that I was using it at all.

This episode was then broken down into smaller codable units known as turns (each turn begins on a new, starred line in the example following this paragraph). The codes that were assigned are in the square brackets following each turn. Turns were coded without a priori labeling, but with discernment for tool use and material or social mediation:

> *Elvira: That's very interesting because you, you know, you said because you didn't really use the blocks at all. [Code/s: researcher probe; using visual manipulatives]
>
> *Caitlin: Yeah, so maybe, you know maybe I was using them [Code/s: using visual manipulatives; tool use by individual; tool use within group work]
>
> *and not even realizing it [Code/s: intuitive tool use]
>
> *(She is speaking while watching clip 3 on the computer)... [Code/s: stimulated recall]
>
> *Yeah, no that was probably definitely...'cause I mean, I can see from…from looking at this (watching the clip and pointing to herself on the computer monitor), [Code/s: stimulated recall; gesture]
>
> *I was definitely using the tools, [Code/s: using visual manipulatives; tool use by individual; tool use within group work]
>
> *maybe I just didn't even realize that I was using it at all. [Code/s: intuitive tool use]

Finally, themes were culled from codes that were identified in the sum data. These themes grew out of all of the coded data turns that were then organized to create categories. Some of the coded turns in the example data above fell into one of the larger themes that eventually emerged from that sum data which was, "Tools mediating group discourse as visual manipulatives/stimuli for collaborative problem-solving."

Results

Simple material artifacts (block manipulatives and paper tools) seemed to both motivate and mediate problem-solving discourse because their particular affordances appealed to the particular problem-solving tasks in ways that more complex tools did not. We identified four dominant categories regarding how tools functioned with regards to the participants' collaborative problem-solving activities. Tools and representations mediated group discourse when they served as: 1) visual manipulatives/stimuli for collaborative problem-solving; 2) a means to explain an isomorphism between the tasks by highlighting correspondence between the constructible towers and the possible pizzas; 3) important elements for the successful employment of a particular learning strategy, process, or behavior; and 4) visual explanations or diagrams for formulated understandings or completed ideas. Although a variety of tools were identified throughout the condensed discourse data, two major material mediators were identified due to their high frequency in the data: blocks (concrete, manipulative tools) and paper tools. Analyses of select examples describing the three dominant categories of material mediation in the problem-solving discourse follow. The examples are ordered as chronologically as possible to chronicle the group's collaborative progress.

Visual Manipulatives

Blocks were used primarily in clips 1, 2, and 3 where the members of Group 5 worked on the first part of the problem: the block towers and pizza topping combinations problems. The group members began their investigations of the towers task by initially constructing as many towers as they possibly could. One strategy that they used to generate these towers was to form "opposite" towers to towers they already constructed: two towers were opposites if placed next to one another because they did not have the same colors at any level (e.g., the opposite of a blue-blue-yellow-blue tower would be a yellow-yellow-blue-yellow tower). In clip 3, it appeared that the blocks mediated the group's solution construction by serving as accessories in real-time, instrumental interaction. That is, the visual appearance of the blocks made forming opposites a natural strategy to employ.

It is interesting to note that the group members believed that the second task, the pizza problem, did not readily lend itself to a visual representation. Although the problems were isomorphic, the group members did not use a strategy analogous to the opposites one they used for the towers. For instance, students did not generate a new pizza from an existing one by including only the toppings that were not present on the other one. This observation is consistent with other students who worked on the pizzas and towers task (Powell, 2003). The opposites strategy may have been directly afforded by the blocks. In fact one student, Caitlin, indicated was not aware of how the blocks were influencing her problem-solving:

> Yeah, so maybe, you know maybe I was using them and not even realizing it (She is speaking while watching clip 3 on the computer)... Yeah, no that was probably definitely...'cause I mean, I can see from…from looking at this (watching the clip and pointing to herself on the computer monitor), I was definitely using the tools, maybe I just didn't even realize that I was using it at all.

Later, Caitlin tried to formulate an algebraic explanation to calculate the total number of block towers based on the number and type of blocks in each tower. As different members of the group introduced different bits of mathematical reasoning into the group discourse, Caitlin tried to direct the discourse towards the mathematical solution that she felt was at the heart of all of their suggestions. She tried to prove an exponential theory that was suggested by directing discourse towards finding a reason as to why a base of two (with an exponent that referred to the number of blocks in a tower, i.e., 2^4) happened to work when calculating combinations. She indicated that the base might have some connection to the two different colors of the blocks in the towers. During the collaborative discourse in this clip, Caitlin gestured and touched the blocks stacked on her desk:

> Caitlin: (gestures towards a block tower on her desk). Oh! Four *squared*. So, you're saying four squared…?
>
> Bob: So it's, it's squared, this is—
>
> Matt: You go over this…and I'm like, there is a reason.
>
> Liz: Yeah, because it's two, because you're using two. Alright, so when you square it, it's not so much the—

Caitlin: But then when you had three cubes, it wasn't…three…squared…

Matt and Caitlin (simultaneously): It was two to the third.

Helen: Yeah, when you have three toppings—

Caitlin: So—

Helen: It's also—

Matt: So the number of cubes is the exponent.

Caitlin: Yeah…and then for this one the same (points to block tower). The two is the base…and n is the number of toppings. But we just don't know why it's two. Yeah, like, I mean it works out both ways but why is it two? I feel…you know what I mean, does two represent a color? Like, I don't know.

In this episode, the group attempted to determine which suggestions they made applied to the combination solution that they had formulated so far. They tried to connect their mathematical solution to the concrete artifacts both in their use of language (e.g., three cubes) to the exponent in 2^3 and their gestures. There is an important difference between how the blocks were used in the construction of towers and in this episode. In the construction of the towers, the blocks served two purposes: they represented members of the set of four-tall towers that could be produced, and they served as cues for producing new towers based on ones that were already produced. In this excerpt, the towers served the communicative purpose of objectifying abstract elements in the mathematical situation. Each of the four block locations represented a choice that could be made, and the two colors represented the number of choices.

Identifying Isomorphisms

In clip 2, the group tried to finalize a mathematical solution that would summarize the findings they had reached with regards to their block and pizza combinations. Some group members realized that 2^n n-tall towers could be formed with yellow and blue blocks and that 2^n pizzas could be created if there were n toppings to choose from. Caitlin, Bob, and Matt appeared to have at least a partial understanding of the algebraic formula behind the block and pizza problems, but Carla, Liz, and Helen had trouble grasping the connection between the algebraic formula and the blocks on the table in front of them. The group attempted to find isomorphisms that would link the surface representations of the blocks and pizzas together. In this way, they tried to transfer a connection across the problems and representations. For example, Matt used blocks to show Carla that there was an isomorphism between the sequence of colors in the block towers and the pizza topping combinations:

Caitlin: So, is it, is it the same thing as this? (Gestures at the blocks on her desk).

Matt: Yeah, it's, it's basically the same thing.

Carla: No, it can't be because there are four different combinations. This is only two different combinations.

Caitlin: No, it can't be.

Matt: I know but, but like, each level represents a different topping, instead of (pauses, points to lowest level of a block tower) like this would represent, like let's say, sausage and this (points to next level up on same block tower) would represent pepperoni and like all the ones with blue on the bottom will have sausage on it, and all the ones without it—

Carla: Oh, I get it. It would be easier if there were four different colors but I get it now.

Prior to this excerpt, a specific stack of four yellow or blue blocks represented a possible tower that could be constructed. In Caitlin's opening remarks, she raised the question as to whether such a stack of blocks could also represent a possible pizza. Matt answered affirmatively and explained how this could be done. Building this explanation required Matt to shift the way that he interpreted the particular tower. In the construction phase, the four blocks represented a *specific* tower, representing a member of the set of the 16 towers that could be constructed. Matt now treated the four blocks as an *abstract representation* of the towers. He appeared to ignore the actual colors in the stack of blocks, attending only to the position of the blocks. He described a general function that would map any 4-tall tower to a unique pizza.

Carla initially rejected the idea that there could be a mapping between the towers and the pizzas, because constructing the towers involved binary choices (yellow or blue) while building a pizza appeared to involve choosing amongst four options (peppers, pepperoni, onions, or sausage). This difficulty has been observed with other students working on the same problems (e.g., Powell, 2003). Carla appears to believe that Matt's explanation resolved her difficulties. However, if the towers could be built from four different colors, the towers and pizza problems would not be isomorphic, and Matt's relationship would no longer hold.

Hence the blocks mediated the conversation in important ways. Caitlin's initial reference to the blocks focused the discussion on whether the blocks could represent a pizza as well as a tower. The blocks played a critical referential role when Matt was describing the mapping between the towers and the pizzas. Matt used the blocks to objectify the notion of the binary choice being made at each level and described how a similar binary choice was being made when deciding whether an individual topping should be placed on a pizza.

Tools/ Affordances for a Learning Strategy

Group 5's discussion in clip 5 centered on the nature of the blocks as potential affordances for a particular type of learning strategy. When clip 5 was recorded, the group had already worked on both the block and pizza problems and had watched a videocase where Brandon, a middle school student, explained how he had solved the same pizza problem using stacking blocks and a chart that he had constructed. The group discussed Brandon's use of blocks in the videocase and to what degree the blocks may have aided Brandon in solving the pizza problem:

> Bob: Are, are…do we need to break into this whole, um...about who gave him the blocks? Is that really—
>
> Matt: I mean…ya know...in the, in the book—
>
> Caitlin: The only reason why we were debating that was because we were trying to figure out, number one, although this may be difficult, we were trying to figure out, if he was a hands-on learner. Like maybe the block problem would be easier because if…it was a hands-on activity.

In this clip, Caitlin repeatedly asked both Cindy and her group whether Brandon was given the blocks or whether he asked for them in the videocase they had viewed. The group's discussion centered on understanding what role tools may have had in mediating Brandon's solution construction. If Brandon needed the blocks to figure out the solution to the pizza problem, then they felt that this would indicate that he was a visual learner and that tool mediation contributed a great deal to his solution of the problem. In pursuing this idea, their own discourse was mediated by the acknowledgment that tools may be necessary to significantly influence learning processes and strategies. In cultivating an appreciation for the effects of material mediation in the Brandon videocase, the group became more reflective about the role of tool mediation in their own learning.

Visual Explanations

Tools also functioned as reference, proof, or justification of the group's final solutions. In clip 6, Bob used a poster, which featured illustrations of several block tower combinations in red and blue and a pizza, as an explanatory tool to visually demonstrate the solutions the group had formulated through their collaborative problem-solving discourse:

> And likewise, when he went back to the um…block problem (gestures at the stacking blocks illustration), he was able to actually understand it further, because he used his information from the pizza problem (moves hand down to the pizza illustration) and said, 'Hey wait...this graph is just the same as these blocks (moves hand back up to the stacking blocks illustration)… So...ah, (reads verbatim from poster) number four, what activities did Brandon use to contribute to his learning…strategies? Ah...in his activities...included the use of tools. Um, (looks at the poster) pedagogical tools which are, ya know, he created a chart to organize his thoughts…So you can say that, the blocks (points to stacking blocks illustration) were actually ah...performance tools...and then um...and then the pedagogical tools which focus on...changing the user's competence ah, example, a stimulation designed to change the literate understanding of math, mathematical concepts…which was the pizza problem (moves hand down to point at the pizza illustration).

As Bob explained Group 5's understandings of the presentation proposal questions, he engaged with the tool in several ways: 1) reading from the poster; 2) gesturing at different aspects of the poster during his explanations; 3) touching the illustrations of the stacking blocks and the pizza; and 4) turning his head to address the student audience, but keeping his body turned and right hand outstretched towards the poster. The poster mediated Bob's performance and presentation as he looked to the poster for 1) a comprehensive account of the group's collaborative solutions, 2) a visual supplement that he could use to demonstrate the correspondence between the block and pizza problems, and 3) the order and delivery of this information to his audience. The poster also served as a supportive prop and visually clarified or reinforced appropriate points from Bob's explanation.

The group's poster is central to this portion of the clip; it mediated not only Bob's behavior, but also directed the visiting students' behavior and attention. The poster served as a benchmark, an indicator of what the group discussed and what solutions they agreed upon. At the same time, the representation served as a stepping-stone to the next level of problem-solving discourse. Hence, here we see what Gravemeijer (1999) would describe as a vertical shift, where a representation of a previous activity becomes an entity on which further mathematics can be done. The group did not have to go back over the material they had covered, except as referential information pertinent to new discussion. Using the poster, Bob was able to transform and re-present aspects of the group's problem-solving experience to create an objective explanatory experience for the visiting students. The existence of the poster mediated the framework of future collaboration in a chronological sense: it punctuated group discussion by serving both as an indicator of completed discussion topics and as stimulation for new discussion topics.

That the poster mediated Bob's performance is evident by his focus on the results of the group discourse, not on their process. When Bob read verbatim from the poster (or from a nearby textbook), he indicated that the information on the poster (as in the textbook) was of a finished, definitive nature: something that served as a reference rather than work in progress. In this way, Bob demonstrated that the group's problem solving had moved from processing to processed.

Discussion

Material Mediation and the Construction of Mathematical Meaning

Different material tools were used in a variety of ways to make or convey meaning in Group 5's knowledge-building process. The members of the group referred to the stacking blocks, paper posters, and technological resources in different ways, all of which indicated different levels of significance for the meaning-making properties each tool afforded.

We found that Group 5 used or referred to the following material mediators most during their problem-solving discourse and collaborative work: block tools (manipulatives) and paper tools. The block tools were used most frequently to mediate (in ranked order): 1) visual explanations of proposed solutions to the problems, 2) in-process, collaborative problem-solving, and 3) explanation of a correspondence across the bock and pizza problems. Paper tools were similarly used to mediate 1) visual explanations of proposed solutions to the problems and 2) in-process, collaborative problem-solving, but less frequently than the block tools.

This study illustrated how manipulatives may enable students to come to recognize important mathematical relationships through the use of tools. In mathematics education, instruction sometimes involves students interacting with manipulatives: physical tools that represent important mathematical relationships in a salient way. For example, the widely used Dienes base-ten blocks provide a physical representation of place value. However, the relationships that are salient in representations to those who are knowledgable of mathematics might not be obvious to someone who is still learning the mathematics (e.g., Cobb, Yackel, & Wood, 1992; Zazkis & Liljedahl, 2004). Indeed, without carefully designed instruction, some manipulatives may be instances of Bereiter's (1985) learning paradox, in the sense that one might need to already understand the mathematics represented in the manipulatives to interact with them in a meaningful way, which would inhibit their pedagogical value.

The blocks used in this study share these characteristics with the manipulatives described above. To someone who understands combinatorics, the physical appearance of a four-tall tower can provide insight into how many four-tall towers can possibly be built. Each of the four levels of the tower represents an independent binary choice. However, this was not obvious to the preservice teachers in this study, nor was it obvious to students who solved similar problems in other environments (Maher & Martino, 1996; Powell, 2003). It was only through the preservice teachers' interactions with the blocks that they were viewed more abstractly. How this process occurred is described in more detail below.

The block and paper tools most frequently served as visual explanations that illustrated the result(s) of the group's problem-solving processes. They also functioned as markers which indicated the current "level" of problem-solving discourse as well as encouraged relative, more complex levels of problem-solving discourse. When the group created a finished poster detailing the results of their collaboration, these paper tools often served as comprehensive, visual agreements as to the nature of the collaborative solution(s) the group advocated. Additionally, these tools indicated fruition in terms of collaboration: posters served as completed, definitive references that chronicled the results of these processes.

The block and paper tools functioned as catalysts for both starting and finishing group collaboration as well as in-process tools with which to collaboratively work through proposed solutions. The presence of the Unifix cubes or a particular poster during collaborative discussion was not only the start of many of the group's discussions, but often directed the conversations that followed. Their language when they discussed the blocks with one another, their gestures when they referenced the blocks, and manner in which they handled the blocks during group collaboration indicated that the group both assumed and reinforced the connection they perceived between their formulative mathematical solutions and the concrete manipulatives. During such social knowledge-building intersections, the physical manipulatives supported group discourse. These manipulative turns within and during verbal discourse, in turn, mediated conversation to a point where the concrete arrangements of the blocks remained the focus of much group discourse thus mediating the group's solution process.

Finally, the group also used block and paper tools to indicate correspondence across the block and pizza problems. For example, the group built particular block tower formations as inspired by paper and pencil lists of pizza topping combinations. By using block and paper tools in these ways, the group not only connected the two problems, but also conveyed their underlying solution to the block tower combinations. The use of the blocks to explain the pizza problem established a correspondence across the problems and made the isomorphisms between them evident. The presence and use of block and paper tools during episodes of collaboration that discussed both problems helped the preservice teachers to establish what they already knew about the two problems, what was still necessary for them to figure out, and what connections were being revealed between the problems as they continued their problem-solving activities.

The Role of Tools and Artifacts in Constructing Solutions

The paper and block tools provided rich affordances for these preservice teachers for developing a solution to the mathematical problems. The tools' affordances included: 1) suitability to the creative nature of the task, 2) usability in terms of comfort levels and previous use of the tools by group members, and 3) availability during group collaboration.

The blocks and paper afforded the students different degrees of creative expression when they tried to explain proposed solutions to the rest of the group members quickly and in the heat of the discussion moment. For six people around a classroom table, it was convenient to gather around a large paper poster that afforded all group members visibility, access, and manipulation. The timely recording of the group's collaborative solutions was easily accessible to all members and may have empowered the group with regards to their group problem solving activities. Seeing a "tangible" list of what they had thought about or accomplished thus far on the group whiteboard may have given the participants the sense that they were making progress in terms of solving the problem. The block and paper tools mediated a discourse environment that worked like a creative rehearsal area where suggested solutions could be quickly and casually explored.

Second, students' past experiences may have also played a role in the choice of simple tools. The students may have gravitated towards these tools because of their conceptions regarding the suitability of particular tools to particular tasks (i.e., block tools are good for solving block problems, and paper tools are good for collaborative brainstorming or presentations). It is also likely that all members had some degree of mastery with such tools, using them in similar settings throughout their entire academic experiences. Thus, they were already familiar and comfortable with some of the affordances that these tools offered in collaborative settings. In addition, these tools may have equalized the participants' contributions to the discourse because their facilities in using simple tools like their hands and fingers (block tools) or a marker (paper tools) were likely quite comparable among group members and could promote group discourse.

Conclusion

These results contribute to a growing body of research demonstrating that simple material tools are important in mediating collaborative problem solving. Despite these important roles for tools and representations, their affordances may not necessarily be realized. How tools are used is determined not only by their mathematical affordances but also the student's past experiences with such tools (Katić, 2005). In this study, we illustrated how students' experiences with the Unifix cubes changed the way they used the cubes in their problem-solving activity. Students first used the tools to help investigate the problem situation. Later, the Unifix cubes, as well as the students' inscriptions, were later transformed into records that served to objectify their activity. We also suggest that providing preservice teachers with the opportunity to critically engage with interesting problems and tools to investigate the problem could be important for helping them understand their future students' tool-related thinking and activity. The prospective teachers in this study developed a meta-level appreciation of how a variety of knowledge-building tools and meaning-making modes could contribute to problem solving. This was illustrated when they discussed how Brandon was using Unifix cubes while watching a videotape of him solving combinatorics problems.

There appear to be several practical implications for teacher education instruction in higher education that can be gleaned from this study. First, students may need to articulate their understandings of the nature and affordances of tool use within a problem-solving framework. Perhaps if teacher educators have a better understanding of how their students believe tools may affect their learning processes, they can create activities that deliberately engage their students in problem-solving activities that encourage expansion of both individual and group knowledge bases through collaborative tool use and experimentation. Simple tools, in addition to being affordable and uncomplicated additions to a classroom, may prove to engage students' collaborative problem-solving skills in ways that create potentially different and transformative learning experiences to compare and contrast to other experiences that already exist in their personal learning histories.

Preservice teachers used tools and representations in important ways to mediate their collaborative discourse as they both engaged in solving combinatorial problems and interpreting a child's reasoning about the same problem. Material tools provided a focus for students to negotiate their understanding and engage in social knowledge construction, thus serving an important mediational role. In other words, the material tools gave students something to talk about and focus on as they both built and represented their evolving understanding. Material mediation can provide support for learners engaged with complex ideas as they work together to build a shared understanding. Studies like this are important in understanding *how* material tools serve this key mediating function. Further work is needed to understand the kinds of experiences learners in general, and preservice teachers in particular, need to use tools and representations as effective mediators of their learning.

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ELVIRA K. KATIĆ, Ph.D. is an associate professor of education at Ramapo College of New Jersey. Her principal research interests include preservice teacher learning and teaching practices, visual and social semiotic research, and emergent technological literacies.

CINDY E. HMELO-SILVER, Ph.D. is associate professor of educational psychology at Rutgers University. Her research interests focus on complex learning include problem-based learning, scaffolding collaborative knowledge construction, and complex systems.

KEITH WEBER, Ph.D. is an associate professor of mathematics education at Rutgers University. His research interests include justification, reasoning, and advanced mathematical thinking.

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